A Dalitz Analysis of $D^+ \rightarrow K^-\pi^+\pi^+$ Decays

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[For the E791 collaboration and in lieu of Brian Meadows.]

Introduction

- The story begins ca. 1987 when Mark III published an analysis of $D^+ \to K^-\pi^+\pi^+$ decays.
- In 1993, E691 confirmed the main features: a strong non-resonant amplitude and a poor fit.
- Also, around 1987, LASS published $K^-\pi^+$ scattering results.
- Around 1996, Bill Dunwoodie suggested that E791 should do a detailed study of $D^+ \to K^-\pi^+\pi^+$ decays to obtain the scattering amplitude in a model-independent way and compare to LASS results.

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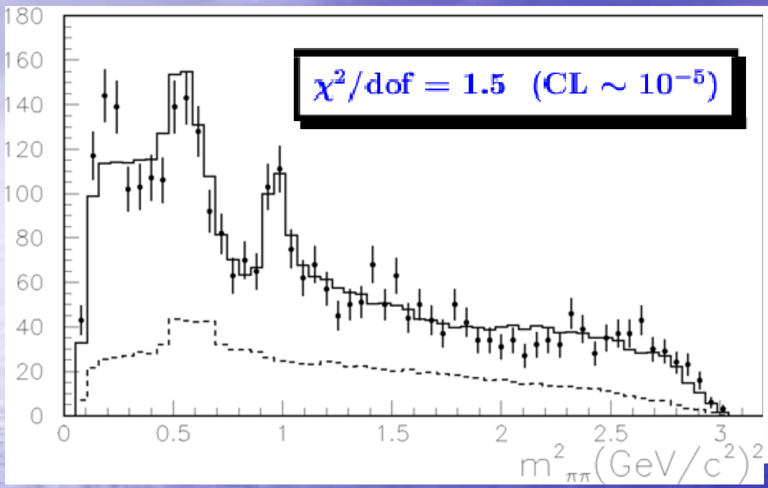
Introduction, contd.

- In the meanwhile, E791 found evidence for a σ in D^+ ! $\pi^-\pi^+\pi^+$ decays and

  a κ in D^+ ! $K^-\pi^+\pi^+$ decays.
- Brian Meadows has now successfully completed an E791 analysis initiated by Bill Dunwoodie's suggestion.
- The paper has been accepted for publication by *Phys. Rev.* D.

E791 D^+ ! $\pi^-\pi^+\pi^+$



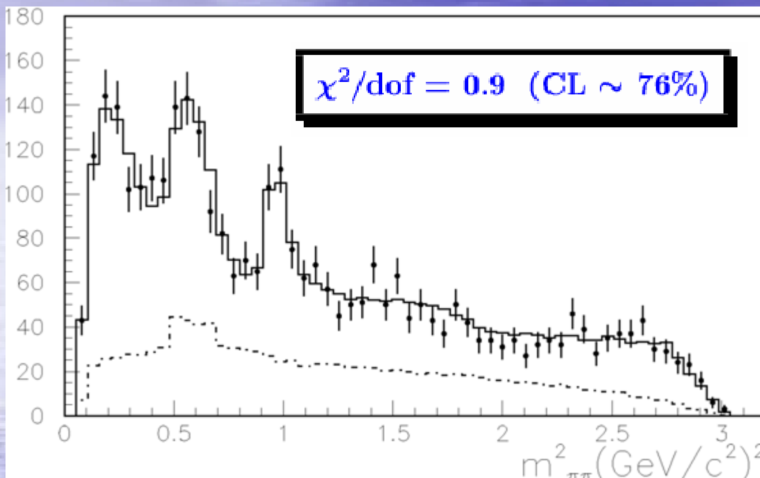
D^+ →

non resonant	$38.6 \pm 1.4\%$	$150 \pm 12^\circ$
$\rho(770)\pi^+$	$20.8 \pm 2.3\%$	0° (fixed)
$f_0(980)\pi^+$	$7.4 \pm 4.3\%$	$152 \pm 16^\circ$
$f_2(1270)\pi^+$	$6.3 \pm 3.3\%$	$103 \pm 16^\circ$
$f_0(1370)\pi^+$	$10.7 \pm 7.7\%$	$143 \pm 10^\circ$
$\rho(1450)\pi^+$	$22.6 \pm 2.1\%$	$46 \pm 15^\circ$

- Non resonant decay is dominant.
- $\rho(1450)$ and $\rho(770)$ next and equally strong.
- Bad fit quality in low mass $\pi^+\pi^-$ region.

No "$\sigma(500)$"

E791 D^+ ! $\pi^-\pi^+\pi^+$



$D^+ \rightarrow$

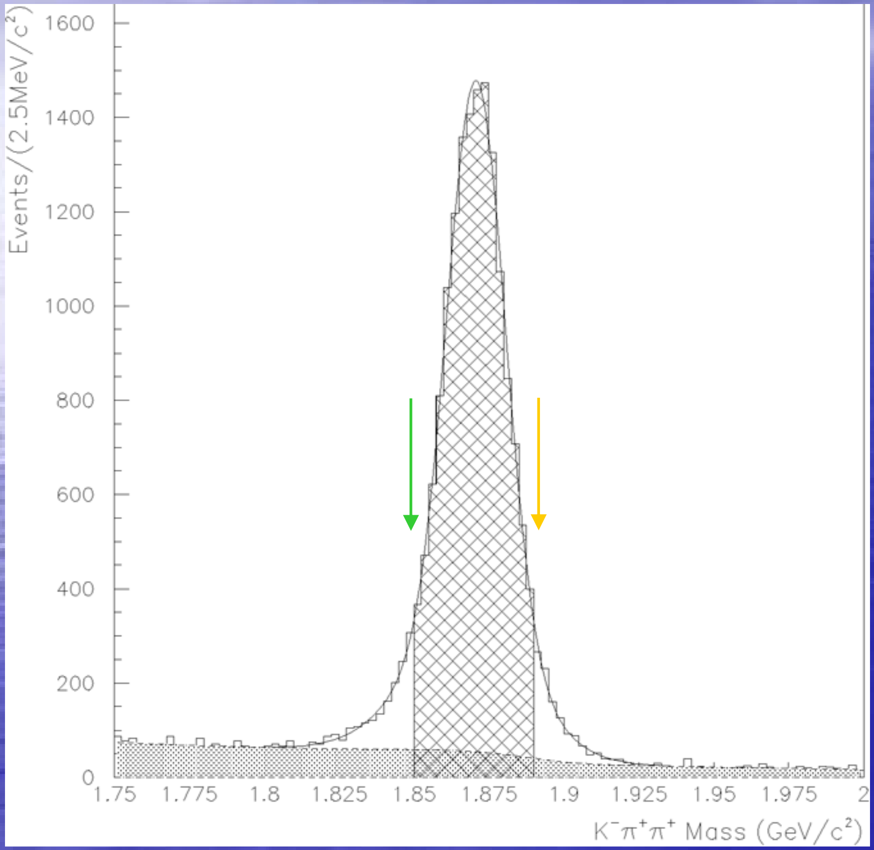
non resonant	$7.8 \pm 6.0 \pm 2.7\%$	$57 \pm 20 \pm 6^\circ$
$\rho(770)\pi^+$	$33.6 \pm 3.2 \pm 2.2\%$	0° (fixed)
$f_\circ(980)\pi^+$	$6.2 \pm 1.3 \pm 0.4\%$	$165 \pm 11 \pm 3^\circ$
$f_2(1270)\pi^+$	$19.4 \pm 2.5 \pm 0.4\%$	$57 \pm 8 \pm 3^\circ$
$f_\circ(1370)\pi^+$	$2.3 \pm 1.5 \pm 0.8\%$	$105 \pm 18 \pm 1^\circ$
$\rho(1450)\pi^+$	$0.7 \pm 0.7 \pm 0.3\%$	$319 \pm 39 \pm 11^\circ$
$\sigma\pi^+$	$46.3 \pm 9.0 \pm 2.1\%$	$206 \pm 8 \pm 5^\circ$

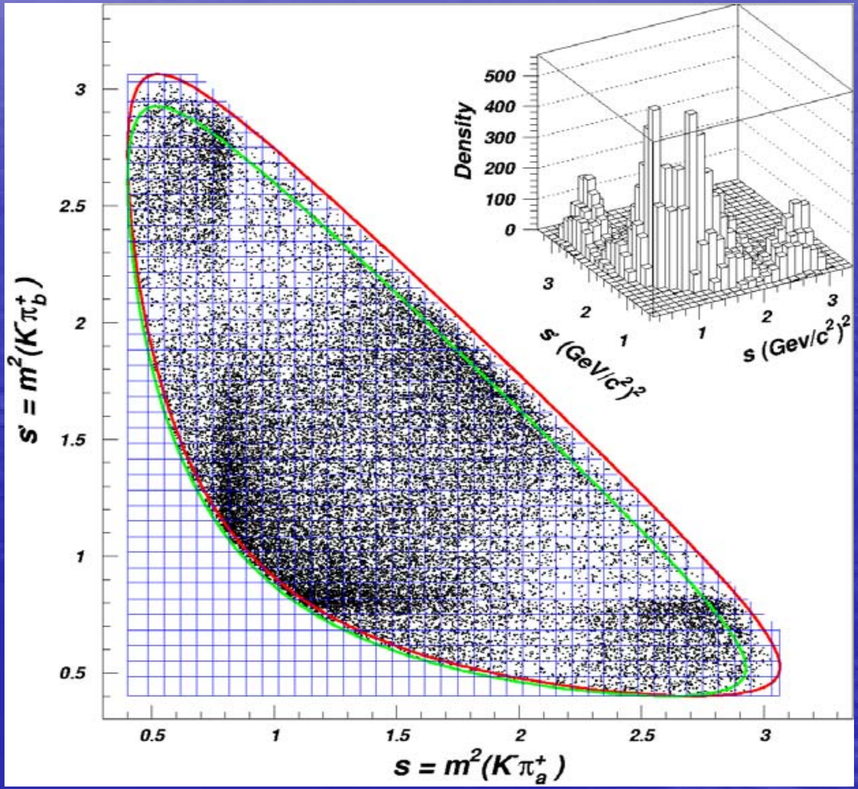
- Model with $\sigma\pi$ has good fit quality.
- $\sigma\pi$ mode dominates the decay - but NR is small.
- $\rho(1450)\pi$ amplitude becomes negligible.

$$m_\sigma = (478^{+24}_{-23} \pm 17)\ \mathrm{MeV}/c^2$$

$$\Gamma_\sigma = (324^{+42}_{-40} \pm 21)\ \mathrm{MeV}/c^2$$

E791 $D^+ \rightarrow K^-\pi^+\pi^+$





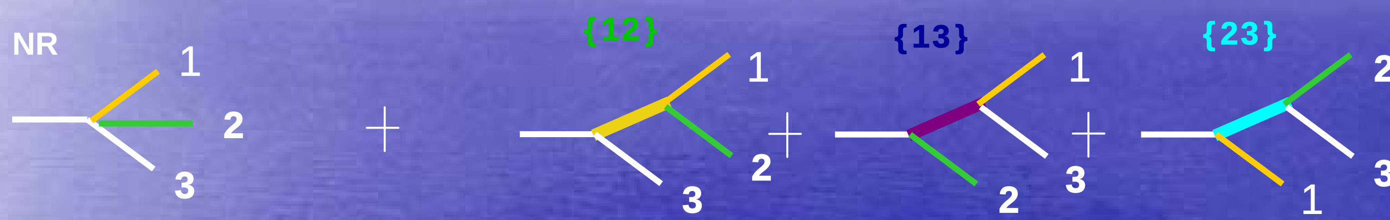
Outline

- The E791 $D^+ \rightarrow K^-\pi^+\pi^+$ Dalitz Analysis:
  - Model-Independent Partial Wave Analysis[1]
  - Comparison with $K^-\pi^+$ scattering results
  - Some comments on related issues
  - Summary

[1]See ArXiv:hep-ex/0507099 – E791 collaboration & W.M. Dunwoodie - accepted by Phys. Rev. D.

“Traditional” Dalitz Plot Analyses

- The “isobar model”, with Breit-Wigner resonant terms, has been widely used in studying 3-body decays of heavy quark mesons.



- Amplitude for channel {ij}:

$$\mathcal{A}_{ij} = d_0 e^{i\delta_0} + \sum_R d_R e^{i\delta_R} F_L^R(p, r_R)\ A_R(s_{ij}) \times F_L^D(q, r_D)\ M_L(p, q)$$

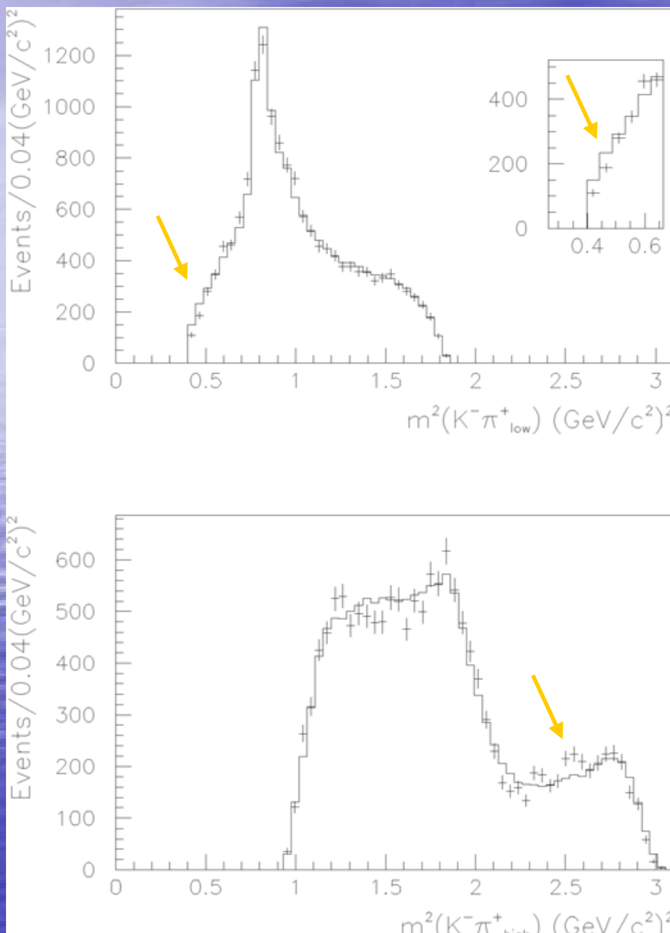
NR Constant — R form factor — D form factor — spin factor

- Each resonance “**R**” (mass M_R, width Γ_R) assumed to have form

$$A_R(s_{ij}) = \left[m_R^2 - s_{ij} - i m_R \Gamma(p, r_R)\right]^{-1}$$

p, q	are momenta in ij rest frame
$r_D,\ r_R$	meson radii

E791 D^+ ! $K^-\pi^+\pi^+$

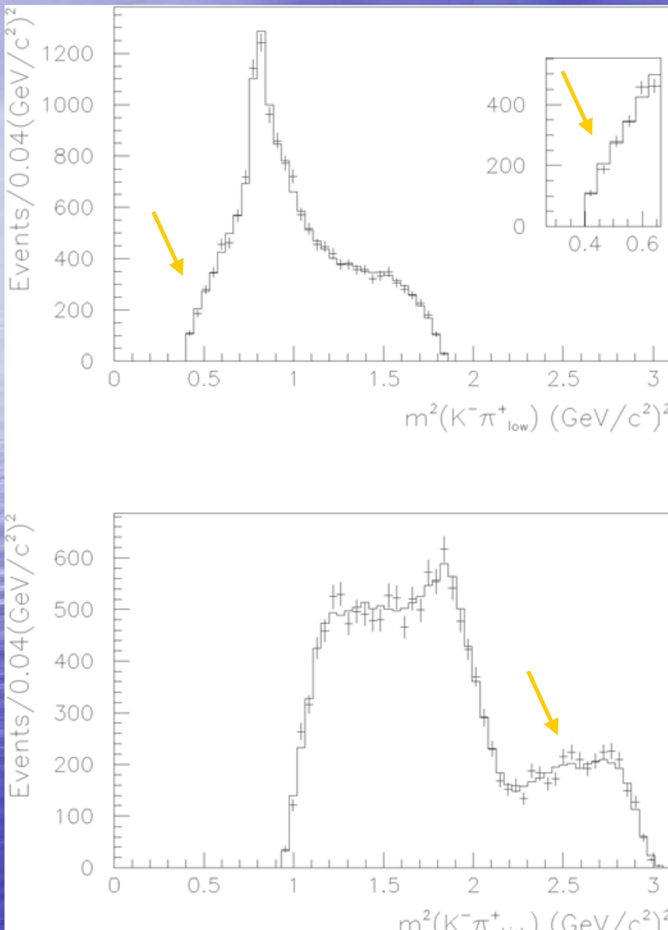


non resonant	$90.0 \pm 2.6\%$	$0°$ (fixed)
$K^*(890)\pi^+$	$13.8 \pm 0.5\%$	$54 \pm 2°$
$K^*_0(1430)\pi^+$	$30.6 \pm 1.6\%$	$109 \pm 2°$
$K^*_2(1430)\pi^+$	$0.4 \pm 0.1\%$	$33 \pm 8°$
$K^*_1(1680)\pi^+$	$3.2 \pm 0.3\%$	$66 \pm 3°$
Total	~138 %	

χ^2/d.o.f. = 2.7

Flat “NR” term does not give good description of data.

κ Model for S-wave



E. Aitala, et al, PRL 89 121801 (2002)

$D^+ \rightarrow$

non resonant	$13.0\pm5.8\pm2.6\%$	$349\pm14\pm8°$
"κ"π^+	$47.8\pm12.1\pm3.7\%$	$187\pm8\pm17°$
$K^*(890)\pi^+$	$12.3\pm1.0\pm0.9\%$	0° (fixed)
$K_0^*(1430)\pi^+$	$12.5\pm1.4\pm0.4\%$	$48\pm7\pm10°$
$K_2^*(1430)\pi^+$	$0.5\pm0.1\pm0.2\%$	$306\pm8\pm6°$)
$K_1^*(1680)\pi^+$	$2.5\pm0.7\pm0.2\%$	$28\pm13\pm15°$)
Total	~89 %	

χ^2/d.o.f. = 0.73
(95 %)
Probability

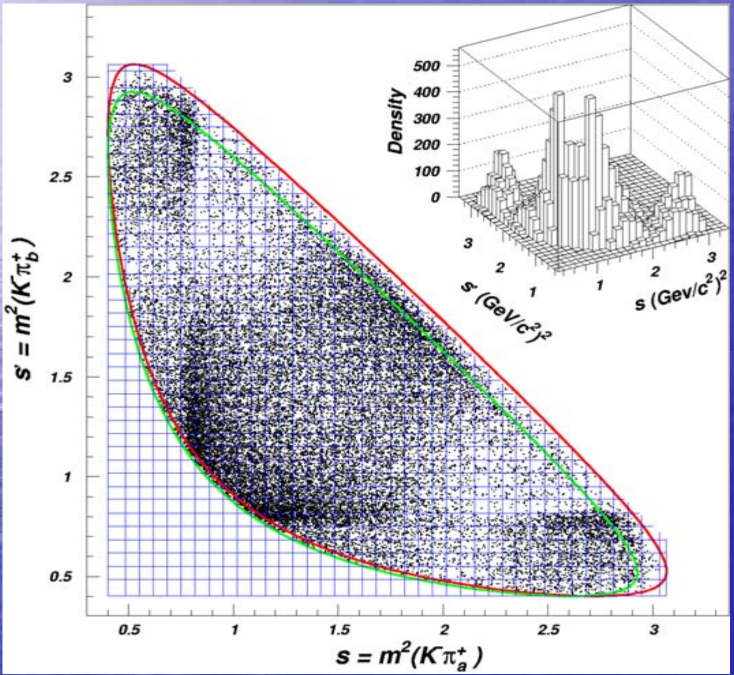
M_κ = 797 § 19 § 42 MeV/c^2
Γ_κ = 410 § 43 § 85 MeV/c^2

Some Comments

- Should the S-wave phase be constrained to that observed in $K^-\pi^+$ scattering (Watson theorem)?
- Are models of hadron scattering other than a sum of Breit-Wigner terms a better way to treat the S-wave[1]
- We decided to measure the S-wave phase (and magnitude) rather than use any model for it.

[1]S. Gardner, U. Meissner, Phys. Rev. D65, 094004 (2002), J. Oller, Phys. Rev. D71, 054030 (2005)

Less Model-Dependent Parameterization of Dalitz Plot



- Prominent feature:
  - $K^*(892)$ bands dominate
  - Asymmetry in $K^*(892)$ bands

  ! Interference with large s–wave component
- Also:
  - Structure at » 1430 MeV/c^2 mostly $K_0{}^*(1430)$
  - Some $K_2{}^*(1420)$? or $K_1{}^*(1410)$??
  - Perhaps some $K_1{}^*(1680)$?
- So

  At least the $K^*(892)$ can act as interferometer for S–wave

  Other resonances can fill in gaps too.

Model-Independent Partial-Wave Analysis

- Make partial-wave expansion of decay amplitude in angular momentum L of produced $K^-\pi^+$ system

$$\mathcal{A}_{ij} = \sum_{L=0}^{2} C_L(s_{K\pi}) \times F_L^D(q, r_D) \times (-2pq)^L P_L(\cos\theta)$$

D form factor

spin factor

- $C_L(s_{K\pi})$ describes scattering of produced $K^-\pi^+$.
  - Related to amplitudes $T_L(s_{K\pi})$ measured by LASS

Model-Independent Partial-Wave Analysis

- Define S–wave amplitude at discrete points $s_{K\pi}=s_j$. Interpolate elsewhere.

$$S(s_j) \equiv C_0(s_j) = c_j e^{i\gamma_j}$$

→ model-independent - two parameters (c_j, γ_j) per point

- P- and D-waves are defined by known K^* resonances

$$\begin{aligned} P(s_{K\pi}) &\equiv C_1(s_{K\pi}) \\ &= F_1^R(p, r_R)\left[BW_{890}(s_{K\pi}) + d_{1680} e^{i\delta_{1680}}\ BW_{1680}(s_{K\pi})\right] \\ D(s_{K\pi}) &\equiv C_2(s_{K\pi}) \\ &= d_{1430} e^{i\delta_{1430}} F_2^R(p, r_R)\ BW_{1430}(s_{K\pi}) \end{aligned}$$

and act as analyzers for the S-wave.

Model-Independent Partial-Wave Analysis

- Phases are relative to $K^*(890)$ resonance.
- Un-binned maximum likelihood fit:
  - Use 40 (c_j, γ_j) points for S
  - Float complex coefficients of $K^*(1680)$ and $K_2^*(1430)$ resonances
    - 4 parameters $(d_{1680}, \delta_{1680})$ and $(d_{1430}, \delta_{1430})$

! 40 x 2 + 4 = 84 free parameters.

Does this Work?

Fit the E791 data:

- P and D fixed from isobar model fit with κ
  - Find $S(s_j)$
- S and D fixed from isobar model fit with κ
  - Find P(s_j)
- S and P fixed from isobar model fit with κ
  - Find D(s_j)

→ The method works.

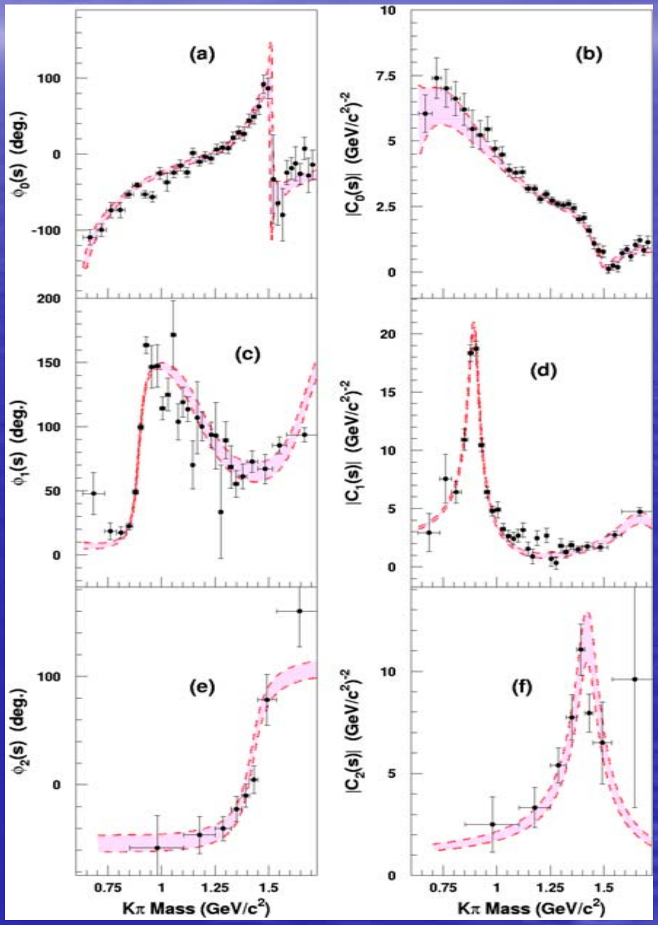
S

Phase Magnitude

S

P

D



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Fit E791 Data for S-wave

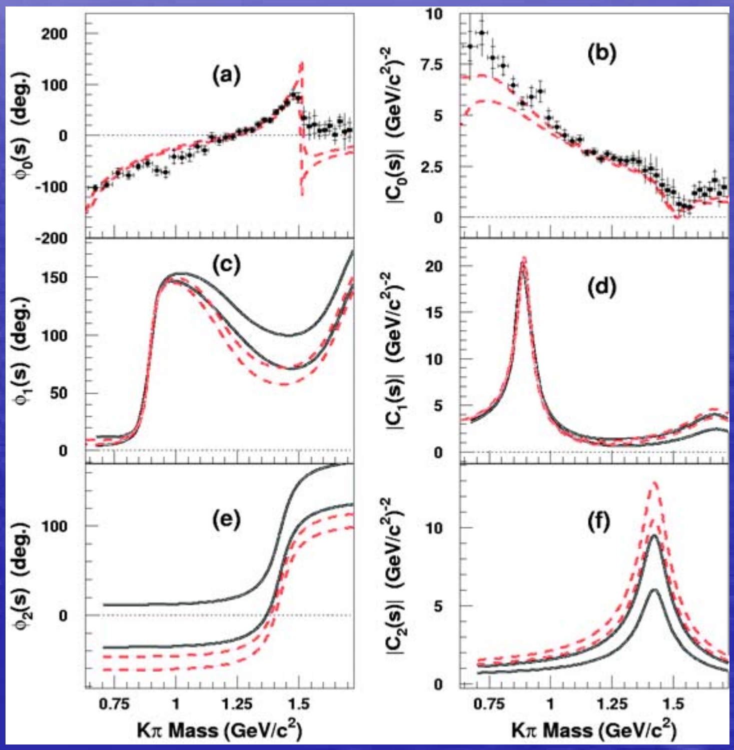
- Find S. Allow P and D parameters to float
  - General appearance of all three waves very similar to isobar model fit.
  - Contribution of P-wave in region between $K^*(892)$ and $K^*(1680)$ differs slightly – balanced by shift in low mass S-wave.

Phase Magnitude

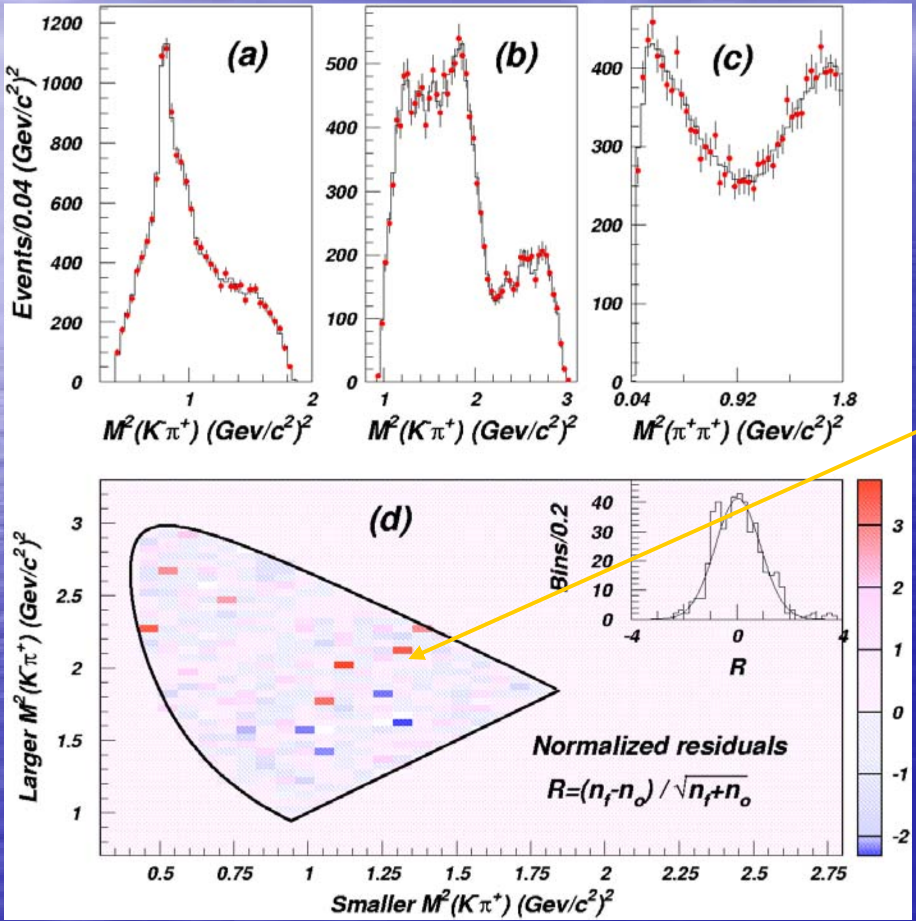
S

P

D



Comparison with Data – Mass Distributions



χ^2/NDF = 272/277 (48%)

Main Systematic Uncertainty

- Even with 15K events, fluctuations in P- and D-wave contributions reflect into S-wave solution.

  Many 15K samples simulated using the isobar model fit from E. Aitala, et al, PRL 89, 12801 (2002)

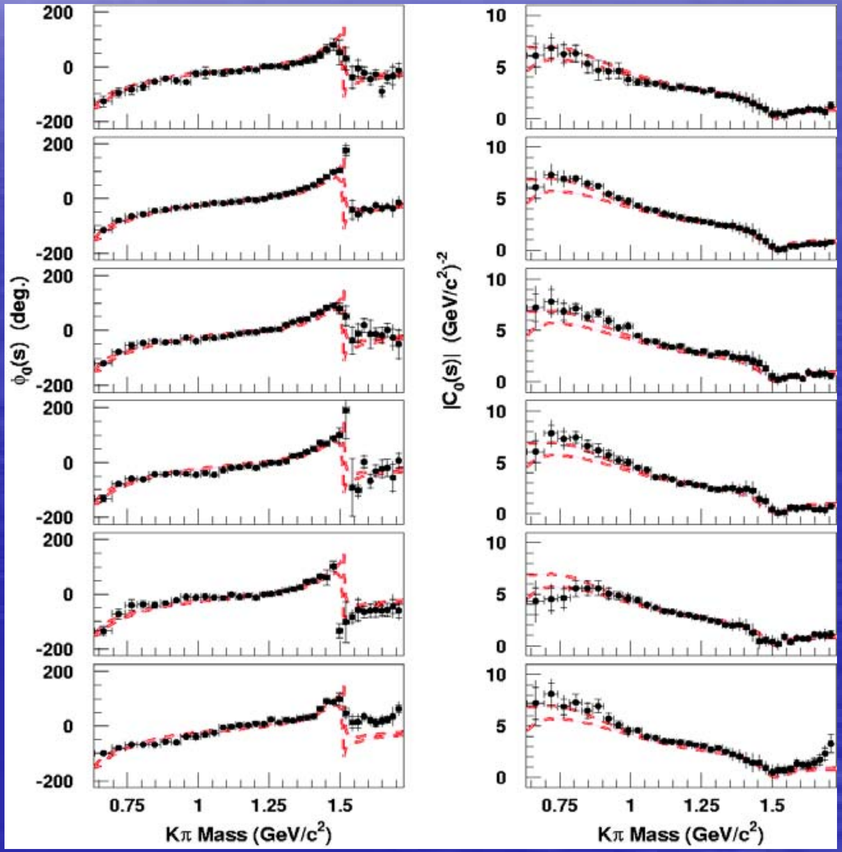
  S

  (first few shown here)

- Solutions similar to those observed in data are common.

S-wave Amplitudes:

Phase **Magnitude**



Another Solution?

- Qualitatively good agreement with data
- BUT does not give acceptable χ^2.
- This solution also violates the Wigner causality condition.

E. P. Wigner, Phys. Rev. 98, 145 (1955)

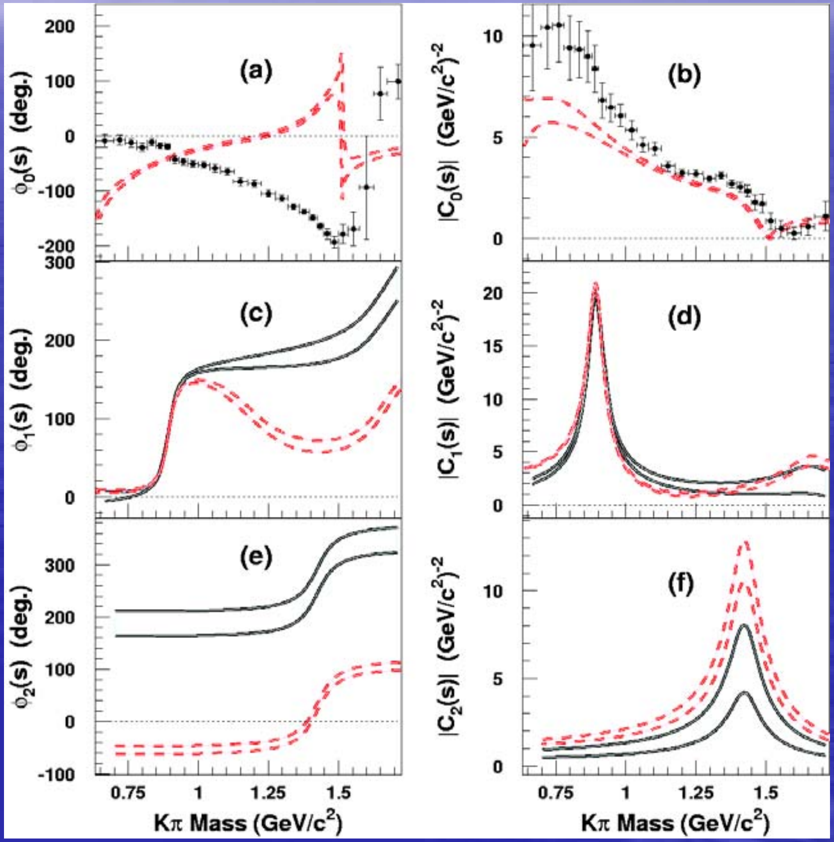
S

Phase | Magnitude

S

P

D



Comparison with $K^-\pi^+$ Scattering (LASS)

- S ($s_{K\pi}$) is related to $K^-\pi^+$ scattering amplitude T ($s_{K\pi}$) :

2-body phase space

Production factor for $K\pi$ system

Measured by LASS

$$S(s_{K\pi})F_D^0 = \frac{\sqrt{s_{K\pi}}}{p} \times \Theta_0(s_{K\pi}) \times T(s_{K\pi})$$

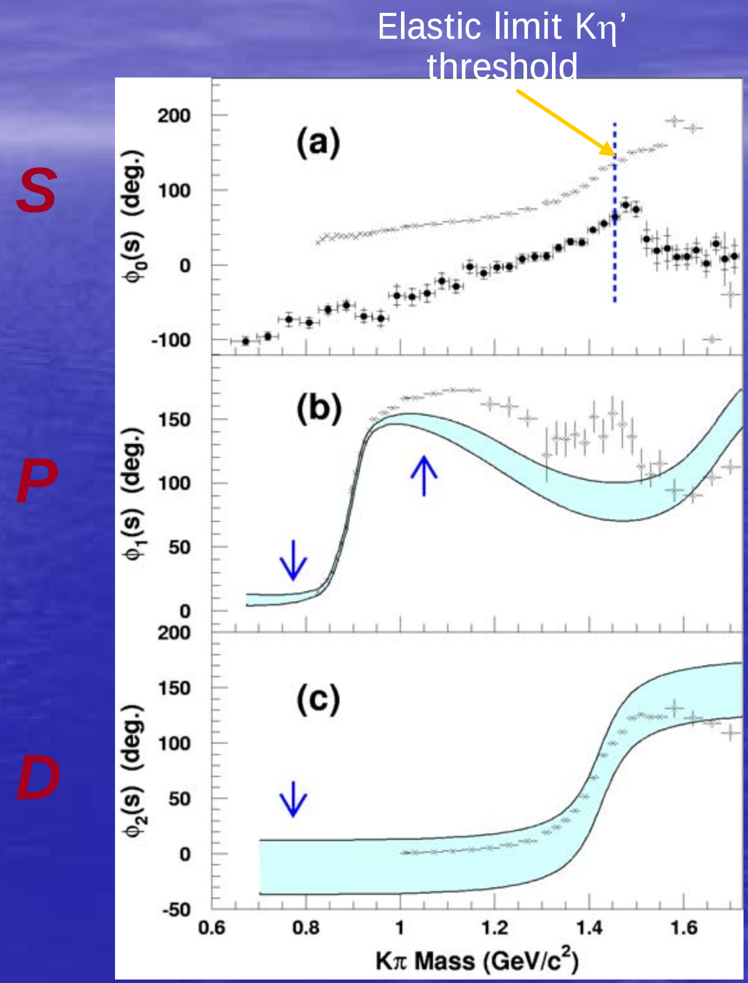
$$T(s_{K\pi}) = e^{i[\gamma(s_{K\pi})-\gamma_0]}\sin[\gamma(s_{K\pi})-\gamma_0]$$

- In elastic scattering $K^-\pi^+$! $K^-\pi^+$ the amplitude is unitary

K.M. Watson, Phys. Rev. 88, 1163 (1952)

Watson Theorem - a direct test

- Phases for *S*-, *P*- and *D*-waves are compared with measurements from LASS.
  - S-wave phase ϕ_s for E791 is shifted by -75^0 wrt LASS.
    - ϕ_s energy dependence differs below 1100 MeV/c^2.
  - P-wave phase does not match well above $K^*(892)$
    - Lower arrow is at $K\pi\pi$ threshold
    - Upper arrow at effective limit of elastic scattering observed by LASS.



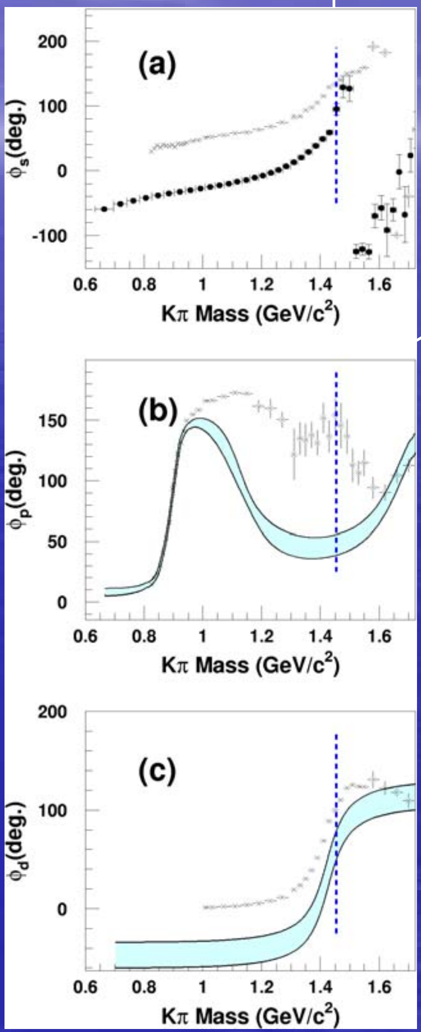
Watson Theorem Enforced for S-wave

- A good fit can also be made by constraining the shape of the S-wave phase to agree with that from $K^-\pi^+$ scattering.
- However:
  - – S-wave phase ϕ_S for E791 still shifted by -75^0 wrt LASS.
  - – ϕ_P match is even worse above $K^*(892)$
  - – ϕ_D phase also shifts.

S

P

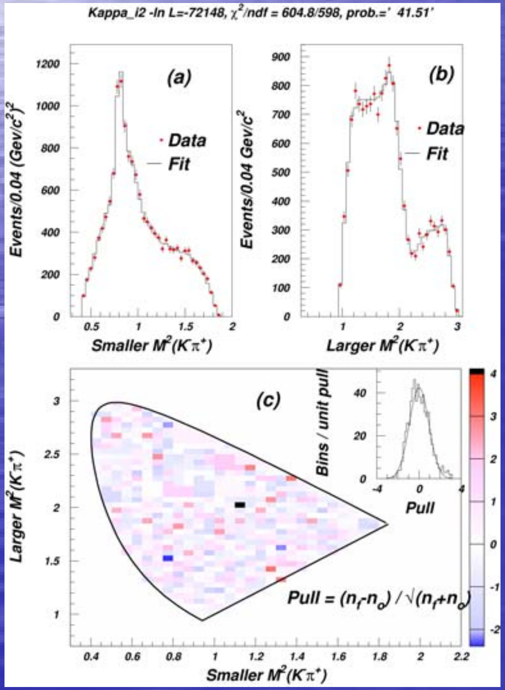
D



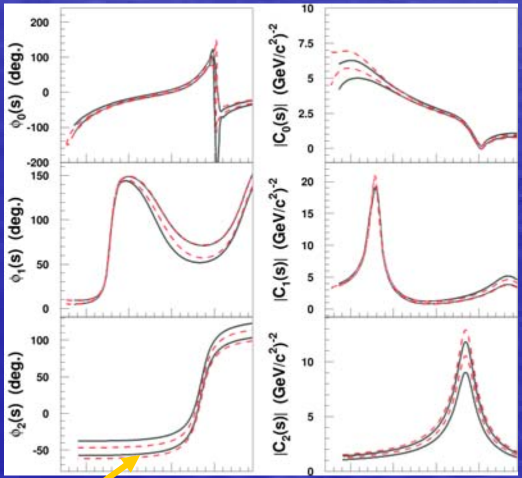
Elastic limit
Kη' threshold
(1454 MeV/c^2)

$\pi^+\pi^+$ (I=2) vs. $K^-\pi^+$ S-wave?

- Add *I=2* amplitude, A_2 to best isobar model fit. An enhancement is known at high mass. (Fit includes a κ isobar):
  - Interpolate phases, $\delta_2(s)$, from Hoogland, *et al., Nucl.Phys.B126:109,1977*
  - Assume amplitude is elastic [$A_2 = a_2 e^{i\alpha_2} \sin\delta_2(s)\, e^{i\delta_2(s)}$]
  - Fit for complex coefficient $a_2 e^{i\alpha_2}$ → Excellent fit



Channel	Fraction %	Amplitude	Phase (degrees)
NR	27.4 ± 8.6	1.58 ± 0.3	-1.1 ± 7.5
$\kappa\pi^+$	32.2 ± 12.7	1.53 ± 0.3	174.6 ± 11.8
$K_0^*(1430)\pi^+$	13.7 ± 1.7	0.58 ± 0.1	50.2 ± 6.4
$K^*(890)\pi^+$	12.2 ± 1.9	1.00 (fixed)	0.0 (fixed)
$K_1^*(1688)\pi^+$	2.7 ± 0.6	2.36 ± 0.4	26.8 ± 8.3
$K_2^*(1420)\pi^+$	0.5 ± 0.2	5.79 ± 0.8	-47.4 ± 9.8
I = 2:		$a_2 =$	$\alpha_2 =$
	0.7 ± 0.7	[illegible].14 ± 0.61	120.2 ± 27.7



- S-wave $K^-\pi^+$ dominates over I=2
- $K^-\pi^+$ amplitudes and "isobar" parameters virtually unchanged

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Summary

- A new technique for analyzing the amplitude describing a Dalitz plot distribution is used in D^+ decays to $K^-\pi^+\pi^+$.
- Could provide model-independent measurements of the complex amplitude of the $K^-\pi^+$ S-wave system, provided a good model for the P- and D-waves is used.
- New measurements for invariant masses below 825 MeV/c^2, down to threshold, are presented.
- No new information on κ(800) from sample this size
- The Watson theorem does not work well with $D^+ ! K^-\pi^+\pi^+$ decays (or there is an I=3/2 admixture).
- I=2 component not needed by fit.
- Better parameterization of P-wave is needed: perhaps the B-factories can do a model-independent measurement of S, P and D waves using their high-statistics data.

Extra Slides

More on the Watson theorem

- Naïve hypothesis is that our Kπ amplitude is the same as that in Kπ scattering. This is supposed to work in the elastic scattering regime.
- However, we are limited by
  - Quantum number conservation (I/O)
  - Production mechanism of Kπ could play a role
  - Environments are different:
    - D decays have an extra meson
    - Scattering has a nucleon
  - Watson states in his paper that the theorem only applies for low mass
- Also, we don't want a spectator, but there is a symmetrization requirement! How do we factorize the amplitude:

$$A(p_K, p_{\pi_1}, p_{\pi_2}) = \frac{\overline{A}(p_K, p_{\pi_1}, p_{\pi_2}) + \overline{A}(p_K, p_{\pi_2}, p_{\pi_1})}{\sqrt{2}}$$

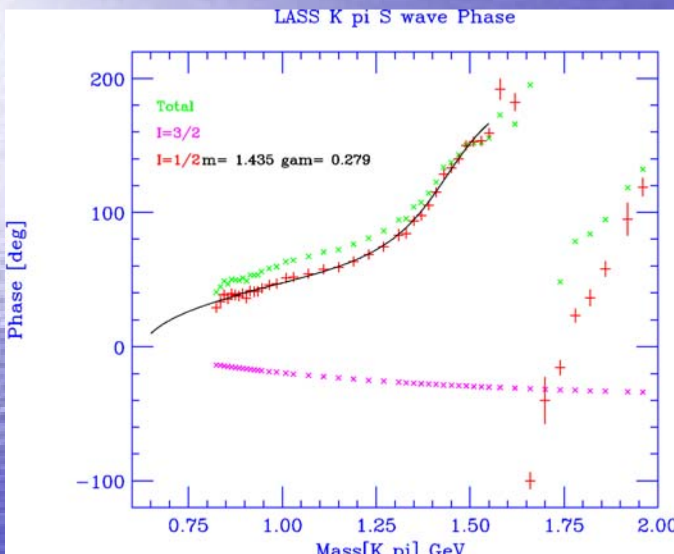
The K-matrix approach

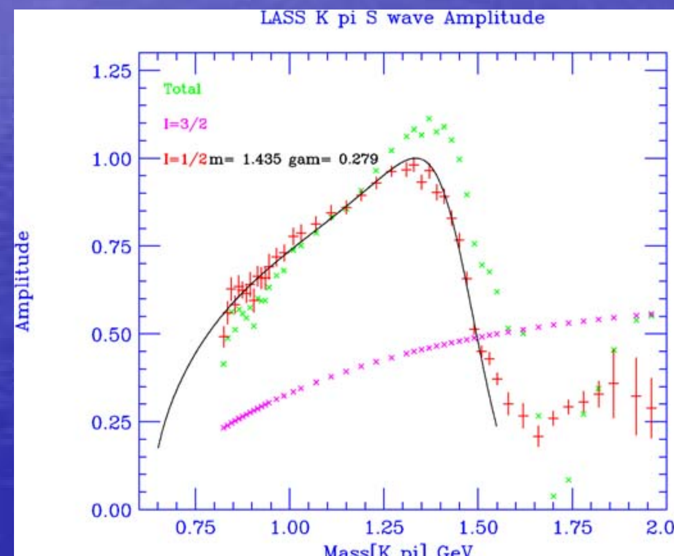
- Most recently championed by FOCUS
- Gives as good result as isobar fit with κ (CL = 7.7% vs. 7.5% for isobar fit. Without a κ, CL = 10^{-6}. See Edera’s talk, Daphne ’04).
- Respects unitarity in $K\pi$ scattering, but is this also true in D decays?
- Further, the K-matrix also requires an ad-hoc parameterization of the non-resonant amplitude.
- Does this mean that the conclusion from the K-matrix work (no broad new scalars are required) is correct?

$K\pi$ Scattering

- Most information on $K^-\pi^+$ scattering comes from the LASS experiment (*SLAC*, E135)

No data from E135 below 825 MeV/c^2





Data from:

K^-p! $K^-\pi^+n$

and

K^-p! $K^0\pi^-p$

NPB 296, 493 (1988)

Parametrize s-wave (I=1/2) by

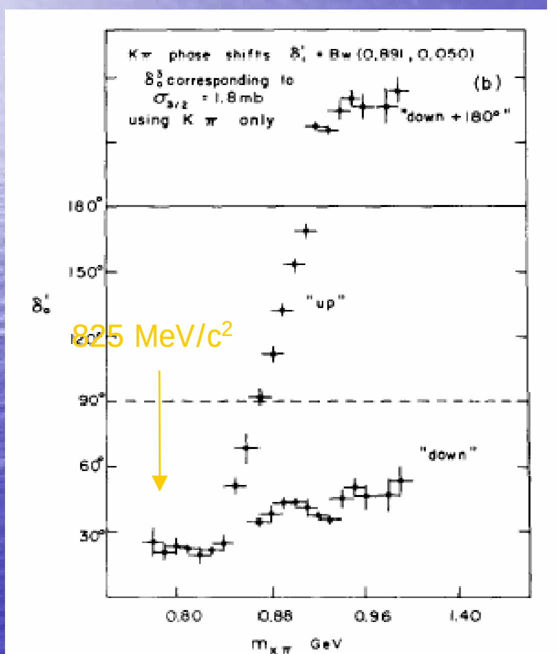
$$
\begin{aligned}
T &= \sin(\delta_R+\delta_B)e^{i(\delta_R+\delta_b)} \\
\cot\delta_R &= m_{1430}\Gamma_{1430}(p)/(m_{1430}^2-s) \\
\cot\delta_B &= 1/a_{1/2}p + b_{1/2}p^2
\end{aligned}
$$

a – scattering length
b – effective range
p – momentum in CM

Kπ Scattering

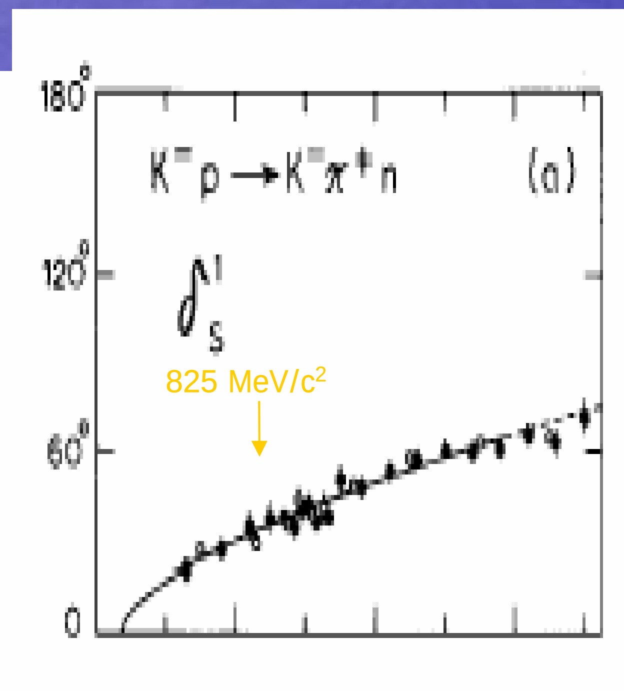
- Relatively poor data is available below 825 MeV/c^2.

I = 1/2

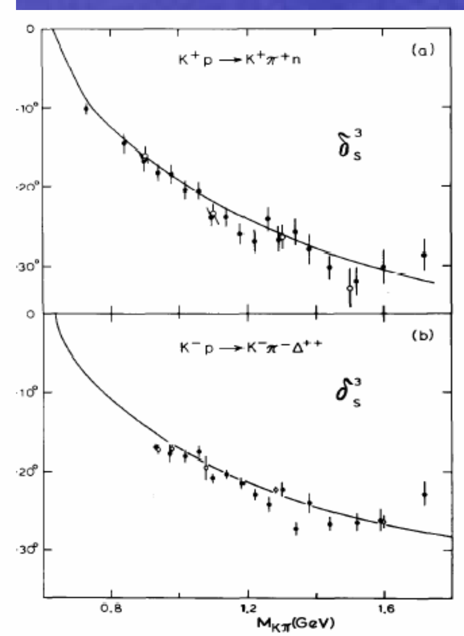


H. Bingham, et al,
NP B41, 1-34 (1972)

I = 1/2



I = 3/2



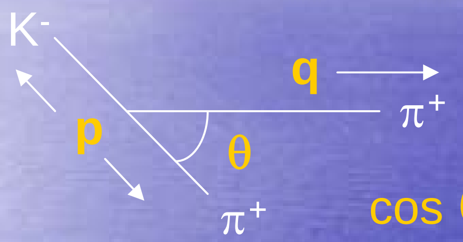
P. Estabrooks, *et al.*, NP B133, 490 (1978)

Kπ Scattering in Heavy Quark Decays

- Precise knowledge of the S-wave Kπ system, particularly in the low mass region, is of vital interest to an understanding of the spectroscopy of scalar mesons.
- It may be possible to learn more from the large amounts of data on D and B decays now becoming available.
- The applicability of the Watson theorem can also be tested.
- E791 is first to use, in this report, a Model-Independent Partial Wave Analysis of the S-wave in these decays to investigate these issues.

Asymmetry in K*(892)

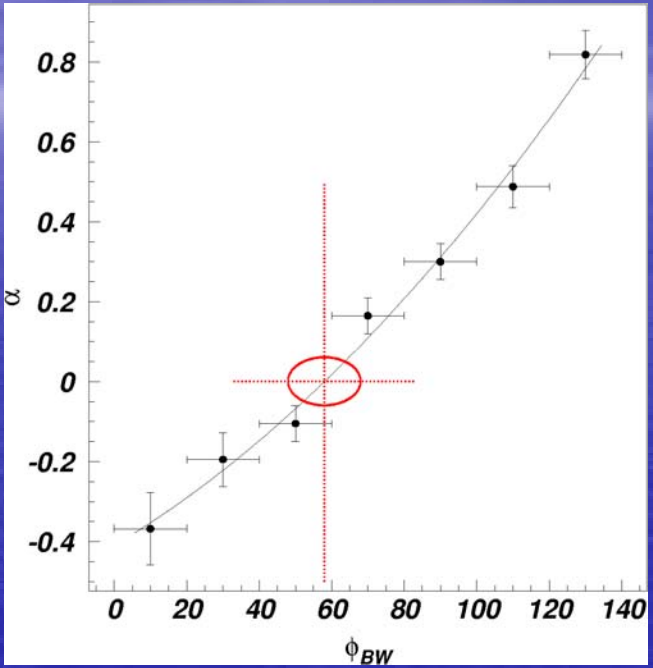
- Helicity angle θ in $K^-\pi^+$ system



$\cos\theta = \hat{\mathbf{p}}$¢$\hat{\mathbf{q}}$

- Asymmetry:

$$\alpha = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)}$$
$$= 0 \quad \text{when } \phi_P - \phi_S = 90^\circ$$

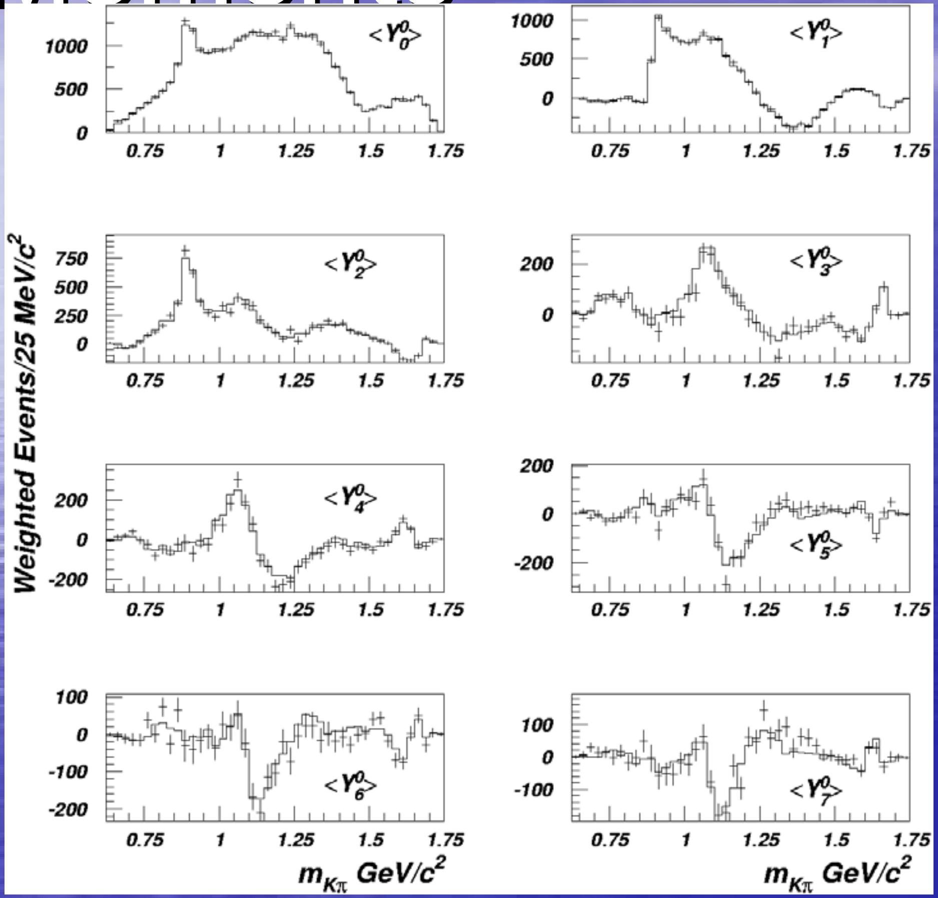


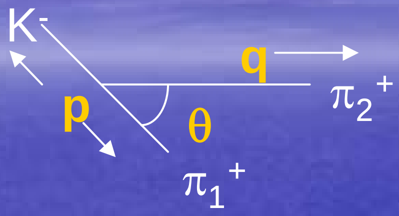
$= \tan^{-1} m_0\Gamma_0/(m_0^2 - s_{K\pi})$ ⟶

LASS finds α=0 when ϕ_{BW}» 135±

! ϕ_P - ϕ_s is -75^0 relative to elastic scattering

Comparison with Data - Moments





- Mean values of $Y_L^0(\cos\theta)$
  - Exclude $K^*(890)$ in $K^-\pi_2^+$

Production of $K^-\pi^+$ Systems

- Production factor Θ_0 ($s_{K\pi}$) is

$$|\Theta_0(s_{K\pi})| = \frac{p}{\sqrt{s_{K\pi}}} \frac{|S(s_{K\pi})F_D^0|}{\sin[\gamma(s_{K\pi}) - \gamma_0]}$$

- Value for γ_0 found by minimizing

$$\chi^2 = \sum_{j=1}^{N_{elastic}} \left(\frac{|\Theta_0(s_j)| - Q}{\sigma(\Theta_0)} \right)^2$$

Production of $K^-\pi^+$ Systems

Plot quantities $\Theta(s_j)$, evaluated at each s_j value, using measured γ_j there.

- Roughly constant up to about 1.250 GeV/c^2

Constant = 0.74§ 0.01 (GeV/c^2)^2.